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An experimental investigation of rheological characteristics of non-Newtonian nanofluids

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ABSTRACT

Rheological characteristics of Al_2O_3, CuO and TiO_2 nano particles were investigated in oil as the base fluid at 1 and 2 wt.%. Constitutive relations for non-Newtonian fluid were discussed based on the power-law model. Measured viscosities of each nanofluid were used to evaluate the power-law and consistency index. Results indicated that the nanofluid viscosity decreased by increasing the concentration. Oil showed shear thickening behavior while nanofluids showed shear thinning behavior. An increase in nano-particle concentration caused a decrease in the power-law index beside an increase in the consistency index. Moreover, the present study showed that the effective viscosity of fluids would be decreased by nanoparticle addition at some wt.% and some shear rates. Furthermore, results showed that the classic models for nanofluid viscosity couldn't predict their real values of nano fluid viscosity, as the measured values are less than the predicted ones.

1. Introduction

Numerous researchers have been investigating better ways to enhance the thermal performance of heat transfer fluids. One of the methods used is to add nano-sized particles of high thermal conductive materials like carbon, metals, and metal oxides into the heat transfer fluid to improve the overall thermal conductivity of the fluid.

Viscosity of a heat transfer fluid is important with respect to studying its convective heat transfer and the pumping power required for practical applications. In industry, an optimization is required between heat transfer capabilities and the frictional characteristics arising from the viscosity of the fluid. Experimental data for the effective viscosity of aqueous nanofluids is limited to certain nanoparticles, such as Al_2O_3 [1-4], CuO [4, 5], TiO_2 [1] and MWCNT [6]. Most of these works have been directed towards metal oxide nanoparticles. The parameters against which viscosity was studied were particle volumetric concentration, temperature, and shear rate. Most of the works have mainly focused on spherical nanoparticles of metal oxides, and on the basis of Einstein theory [7]. Viscoelastic-fluid-based nanofluids with dispersion of copper (Cu) nanoparticles in viscoelastic

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surfactant solution were investigated by Yang et al. [8]. They found that the viscoelastic-fluid-based Cu nanofluid shows a non-Newtonian behavior in its viscosity, and the viscosity increases with a reduction in the temperature. Ravi Prasher et al. [9] performed some experiments and gave results on the viscosity of alumina-based nanofluids reported for various shear rates, temperature, nanoparticle diameter, and nanoparticle volume fraction. From their data it seems that the increase in the nanofluid viscosity is considerable in comparison with the enhancement in the thermal conductivity reported in the literature. Lee et al. [10] studied the viscosity of SiC-water nanofluids and showed that the viscosity of nanofluids increases along with increased concentration and decreased temperature. Utomo et al. [11] investigated thermal conductivity, viscosity, and heat transfer coefficient of titania and alumina nanofluids. Their results illustrated that the viscosity of alumina and titania nanofluids was higher than the prediction of Einstein–Batchelor model. The rheological properties of titania nanofluids with different base fluids were studied by Chen et al. [12, 13]. They found that the titania-ethylene glycol nanofluids showed a Newtonian behavior, while the titania-water showed a non-Newtonian behavior. These finding show that the base fluid has a strong effect on the rheological behavior. Experimental results on the viscosity of the nanofluid prepared by dispersing alumina nanoparticles (<50 nm) in commercial car coolant were reported by Kole and Dey [14]. Their results showed that the pure base fluid displayed Newtonian behavior over the measured temperature, but it transformed to a non-Newtonian fluid with addition of a small amount of alumina nanoparticles. Viscosity of novel copper oxide coconut oil nanofluid was studied by Rashin and Hemalatha [15]. Experimental viscosity studies have shown that most of nanofluids have a non-Newtonian behavior. Rheological characteristics of non-Newtonian nanofluids were investigated by Hojjat et al. [16]. Results showed that nanofluids as well as the base fluid exhibit pseudoplastic (shear thinning) behavior. They found that the apparent viscosity of nanofluids decreased by increasing shear rate and increasing volume fraction.

The objective of the present study is to investigate the rheological behavior of some nanofluids. In few studies it was found that the viscosity of the fluid could be decreased by nanoparticle addition at some vol% and shear rates [17]. The present study was conducted to prove the idea. The power-law model was used to model the non-Newtonian behavior of the fluids. Two fitting parameters, the power-law index and the consistency index, have been calculated for different nanofluids in several concentrations. The base fluid used throughout this investigation was the heat transfer oil. Al2O3, TiO2 and CuO nanoparticles were used in the nanofluids at 1 and 2 wt.% concentrations. The results are compared with classical models, and their discrepancy is discussed.

2. Experimental procedure

Nanofluids are made form Al_2O_3, TiO_2 and CuO nanoparticles of weight fractions 1% and 2% using the two-step method. Transmission electron microscope (TEM) image of CuO nanoparticles is shown in Fig. 1. The heat transfer oil used in this study was purchased from Behran Oil Company. According to ASTM D-1290 standard the density of the above mentioned oil at the temperature of 15.6 ^oC is 870 Kg/m^3 .Since these suspensions are not stable systems, stabilizing methods should be utilized to gain more stable suspensions. In the present study an appropriate amount of SDBS surfactant, Sodium Dodecyl Benzene Sulfonate, recommended by [18] was added to the mixture. Then, the solid particles deagglomerated by intensive ultrasonic device (Ms Hielscher model UP400S) for 3 h after mixing with the base fluid by magnetic forced agitation. In order to prevent fluid from vaporizing, samples remained in mixture of ice

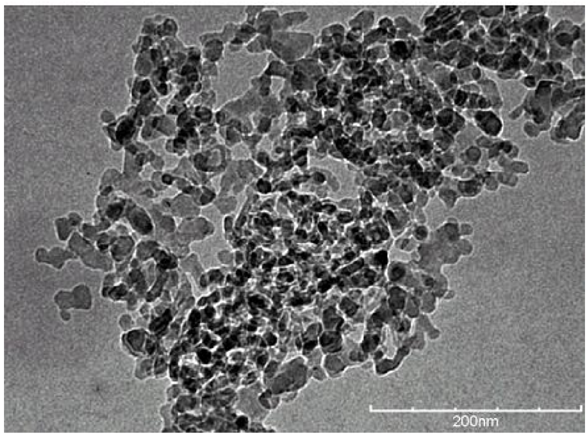

Figure 1. Transmission electron microscopic image of CuO particles with a nominal size of 40 nm

Table 1 Physical property of nanoparticles.

Particle	mean diameters (nm)	Nanoparticle's Purity	Shape of particles
CuO	40	>99%	Spherical particles
Al_2O_3	15	>99%	Spherical particles
TiO_2	20	>99.9%	Spherical particles

and water during sonication. Nanofluids produced were found to be very stable and there was no visually observable sedimentation after more than a week. Table 1 contains other properties of the nanoparticles.

Viscosity of the nanofluids was measured by Anton Paar rheometer (model: MCR-301, figure 2). The viscometer drives the spindle immersed into the sample holder containing the test fluid sample and provides a rotational speed that can be controlled to vary from 0.01 to 250 rpm. The viscometer has been tested and calibrated with the calibration fluid provided by Brookfield Engineering Laboratories. Meanwhile, all the measurements are performed under steady state conditions. Each experiment was repeated three times. The overall standard deviation of the results was below 6%.

3. Results and discussions

Figure 3 shows the variation of apparent viscosity with respect to the strain rate (s^{-1}). It can be seen that the viscosity of the base fluid (heat transfer oil) increases with an increase in the strain rate. On the contrary, the viscosity of nanofluid decreases with an increase in the strain rate by adding the nanoparticles (Al_2O_3, TiO_2, and CuO) to the base fluid. So, the base fluid (heat transfer oil) is a shear thickening (dilatant) fluid while the nanofluid, based on the heat transfer oil, is a shear thinning (pseudo-plastic) fluid. In other words, the behavior of fluid changes by adding the nanoparticles.

The slope and y-intercept of profiles in figure 3 indicate the Power-law index and consistency index of power-law model according to equation 1 [16, 19-23]:

$$\mu = r\dot{\varepsilon}^{n-1} \tag{1}$$

Figure 2. Anton paar rheometer

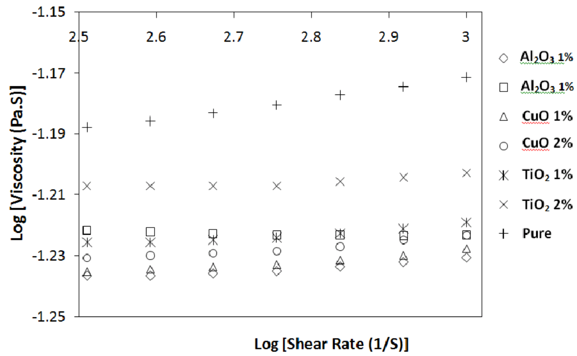

Figure 3. Logarithmic variation of the measured viscosities of the fluids

μ is apparent viscosity and $\dot{\varepsilon}$ is shear rate. r and n are the constants of power-law model. Figure 4 shows the linear behavior of the measured viscosity of the base fluid. From the slope and y-intercept of the fitted line in figure 4, the Power-law index and consistency index of the base fluid can be obtained which are 1.032 and 0.053, respectively. This procedure could be done for the Al_2O_3–oil nanofluid at 1 and 2 wt% which is presented in figure 5.

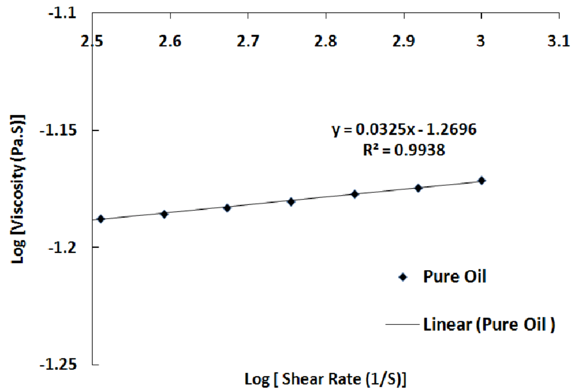

Figure 4. Variation of viscosity of the pure base fluid (heat transfer oil) with respect to the imposed strain rate

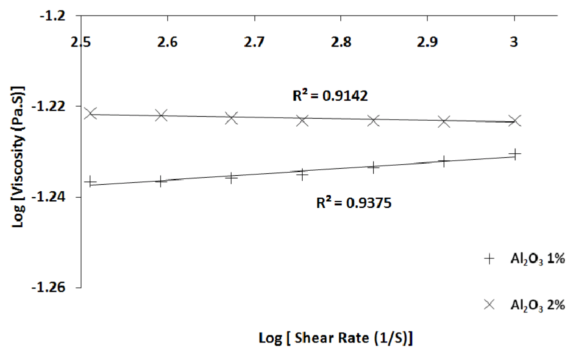

Figure 5. Variation of viscosity of the Al_2O_3-oil at 1 and 2 wt.%

The Power-law index and consistency index of Al_2O_3-oil, CuO-oil, and TiO_2-oil are presented in figures 6 and 7. The non-Newtonian behavior of nanofluids changes with nanoparticle types and wt%. Figure 6 depicts that the power-law index of the fluids decreases with the presence of nanoparticles. This reduction in the power-law index boosts when the nanoparticle wt% increases. Also, the slope of reduction in the power-law index is the highest for the Al_2O_3–oil nanofluid. The power-law index of pure oil is 1.032 and reaches the value of 0.9751 at 2 wt% concentration of Al_2O_3 nanoparticles, while the power-law index of the TiO_2–oil and CuO–oil nanofluids are 0.9906 and 0.9986 at the same wt%.

An increase in the consistency index of the fluids with the presence of nanoparticles is illustrated in figure 7. The maximum increase in the consistency index is for the Al_2O_3 nanoparticles the same as what could be seen for the power-law index in figure 6. The consistency index increases 4, 14 and 20% at 2 wt% concentration for CuO-oil, TiO_2-oil and Al_2O_3-oil nanofluids, respectively.

The ratio of dynamic viscosity of nanofluid to the base fluid is shown in figures 8 and 9 at 1 and 2 wt% concentration of nanoparticles. This ratio decreases with an increase in the strain rate. Al_2O_3-oil nanofluid has the maximum tendency in reduction in the viscosity. The dynamic viscosity ratio decreases about 12% at 1 wt% of Al_2O_3-oil nanofluid while this reduction is about 8% at 1 wt% of TiO_2-oil and CuO-oil nanofluids, respectively. Moreover, the slope of reduction in the dynamic viscosity ratio for the Al_2O_3-oil is the highest. The reduction in the viscosity ratio is 10% for the CuO-oil nanofluid at 2 wt% concentration. This reduction for the TiO_2-oil and Al_2O_3-oil nanofluids is only about 7%. Reduction in the viscosity of the fluid with an increase in the strain rate shows a shear thinning behavior. This shear-thinning behavior is due to the nanoparticle agglomeration clusters which break down when the shear rate is increased and, as a result, the apparent viscosity decreases [16]. Another reason in the reduction of viscosity is the abruption in the base fluid structure. Nanoparticles could break off the connection between the base fluid molecules and act as a connector between fluid layers. Also, Brownian nature of particles and its differences form the Brownian characteristics of the base fluid could be the other reason. These differences force fluid and nanoparticles to move relatively. As a result of this relative motion, the connection between fluid layers and also between the fluid and nanoparticle is continuously constructed and broken off.

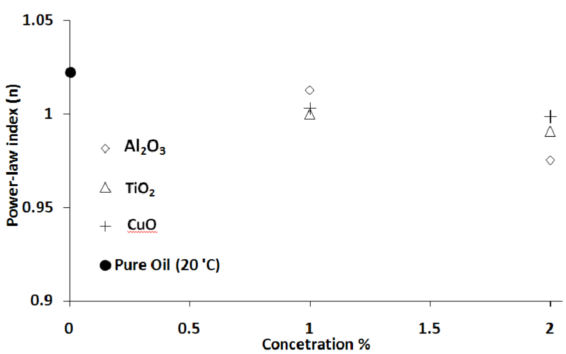

Figure 6. Effects of nanoparticle concentration on the power-law index

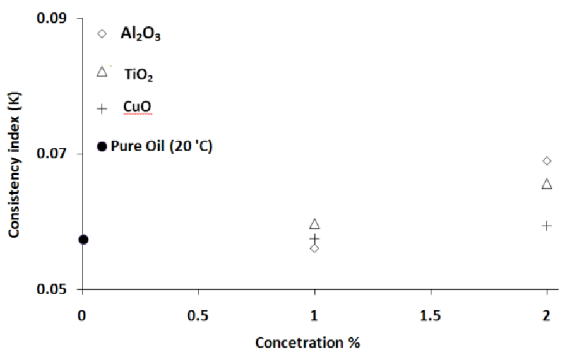

Figure 7. Effects of nanoparticle concentration on the consistency index

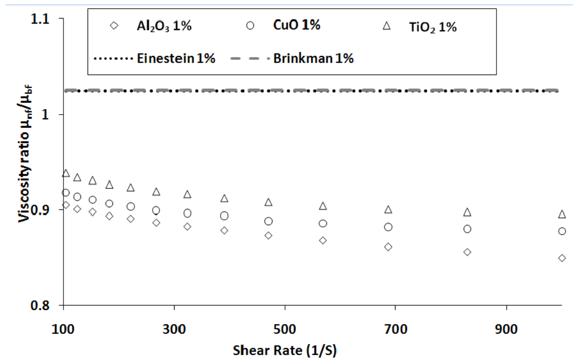

Figure 8. Effects of nanoparticle addition and shear rate on the viscosity of the nanofluids (at 1 wt.% concentration)

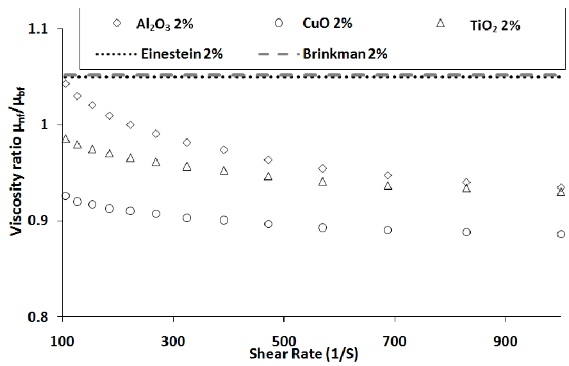

Figure 9. Effects of nanoparticle addition and shear rate on the viscosity of the nanofluids (at 2 wt.% concentration)

The correlations of Brinkman [24] and Einstein are used for comparison. Einstein proposed correlation (2) for the suspensions of solid particles in fluids. In 1999 Drew and Passman [25] showed that the Einstein's relation can be used for the mixtures of micrometer particles with low (<2) volumetric concentrations.

$$\frac{\mu_{eff}}{\mu_{bf}} = 1 + 2.5\phi \tag{2}$$

Brinkman [24] modified the Einstein relation to be applicable in the denser mixtures:

$$\frac{\mu_{eff}}{\mu_{bf}} = \frac{1}{(1+\phi)^{2.5}} \tag{3}$$

None of the correlations can predict the experimental results in figures 8 and 9. At 1 wt%, all of the nanofluids have a lower value for the dynamic viscosity ratio than the correlations (2) and (3). At 2 wt%, the Al_2O_3-oil nanofluid has values the same as the correlations at low shear rates. But, at high shear rates, the experimental results fall below the prediction of the correlations. Moreover, the discrepancies of the experimental results and the correlations are a function of shear rate. It seems that further studies on the viscosity of nanofluids are necessary to develop a correlation for the behavior of non-Newtonian nanofluids and maybe for the reduction in the viscosity. Also, this study shows that the viscosity of nanofluids could be lower than the base fluid at some concentrations and shear rates, on the contrary of all available correlations. The reduction in the viscosity is not a new finding. This behavior was not seen in the previous literatures except for some few investigations like Kole and Dey [17]. But, it wasn't emphasized that the reduction in the fluid viscosity at some vol% of nanoparticle presence occurred in the study of Kole and Dey [17]. Also, the available correlations proposed for the effective viscosity cannot predict the reduction of the viscosity in presence of nanoparticles. However, the reduction in the effective viscosity has very important applications, especially in lubrications and pumping power. The lower the value of viscosity, the lower would be the loss of energy.

4. Conclusion

The aim of the present work was the study of viscosity of some nanofluids based on the non-Newtonian power-law model. The heat transfer oil was selected as the base fluid. Al_2O_3, TiO_2, and CuO nanoparticles were added to the base fluid at 1% and 2% weighted-average concentrations. The highlights of the study are stated as follows:

- The base fluid (heat transfer oil) had a shear thickening behavior. The viscosity of the pure oil increased with increased strain rate, while the viscosity of the nanofluids showed a shear thinning behavior. The viscosity of nanofluids decreased with an increase in the shear rate. So, the nature of the fluid changed with nanoparticle addition.
- Nanoparticles decreased the power-law index of the fluid. The Al_2O_3 nanoparticles have stronger effects on the power-law index than others. It was seen 5% reduction we seen in the power-law index at 2 wt% of Al_2O_3 nanoparticles.
- The consistency index of fluid increased by nanoparticle addition. The Al_2O_3 nanoparticles increased the consistency index about 20%, while the increase of the CuO and TiO_2 nanoparticle is below than 14%.
- The dynamic viscosity ratio of nanofluid to the base fluid is smaller than one for these cases. It needs further studies to indicate at which condition the viscosity would decrease. Also the available correlations proposed for the effective viscosity cannot predict the reduction of the viscosity in the presence of nanoparticles.

The results showed that the effective viscosity of nanofluid could be reduced at some wt% and strain rates. The reduction in the viscosity by the use of nanoparticle would have important applications in lubrication and pumping. By reducing the viscosity of the fluid, the energy loss would decrease.

Appendix

The constitutive relation for the stress tensor of the fluids is presented in this section. The balance of momentum gives [19]:

$$\rho\left(\frac{\partial v}{\partial t} + \nabla v . v\right) = \nabla . S + \rho M + \rho g \tag{1}$$

v is velocity vector, ρ is density, S is the Cauchy stress tensor, M is body force, and g is gravitational force. Based on the nature of the fluids, the Cauchy stress tensor is divided into two parts; deviatoric and spherical parts [20 - 26]:

$$S = dev(S) + sph(S) = \tau + \frac{1}{3} S_{ii} I = \tau - PI \tag{2}$$

I is the unit tensor, τ is the shear stress tensor, and $-\frac{1}{3}S_{ii} = -\frac{1}{3}tr(S)$ is the pressure of the fluid. The Reiner-Rivlin theory is used in defining the Cauchy stress tensor. Form the point, the Cauchy stress tensor is a function of the rate of deformation tensor (D) [23, 24]:

$$S = \phi_o I + \phi_1 (II_D, III_D) D \tag{3}$$

ϕ_o and ϕ_1 are material functions (or material coefficients). The rate of deformation tensor (D) is the symmetric part of the gradient of velocity vector and is defined as follows:

$$D_{ij} = sym(\nabla v) = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \tag{4}$$

ϕ_o has a constant value and I_D, II_D, and III_D are the invariants of the D [23, 24]:

$$\begin{aligned} I_D &= tr(D) = D_{ii} = 0 \text{ (from continuity)} \\ II_D &= 0.5\left[(tr(D))^2 - tr(D^2)\right] = -0.5\,tr(D^2) \\ III_D &= \det(D) \end{aligned} \tag{5}$$

The undefined coefficient (ϕ_o and ϕ_1) should be defined. A comparison between relations 2 and 3 gives that the $-\phi_o$ is the pressure of the fluid. Through experimental investigations it was found that the dependency of ϕ_1 to the third invariant of the rate of deformation tensor (III_D) is weak. In other words, ϕ_1 is a function of the second invariant of D (II_D). Form the second law of thermodynamic it holds that ϕ_1 should have positive values [27, 28]. So, one can write:

$$S = -pI + 2\gamma(-4II_D)D \tag{6}$$

The second invariant of D has a negative value, on the other hand ϕ_1 is a positive function, and regarded to the experimental investigations the equation (6) could be rewritten in the form of equation (7). Fluid which follows this relation is called generalized Newtonian fluids (incompressible).

$$S = -pI + 2\gamma(-4II_D)D \tag{7}$$

γ is a tensor function and modeling should be used to determine it. The common model is the power-law model which is used in the present study. In the power-law model, γ has the following form [23-30]:

$$\gamma = r(-4II_D)^{n-1/2} \tag{8}$$

in which n is power-law index and r is consistency index. Therefore, the Cauchy stress tensor will be [23, 24, 30]:

$$S_{ij} = -p\delta_{ij} + 2r\left|4II_D\right|^{n-1/2} D_{ij} \tag{9}$$

The values of n in the constitutive relation (9) define the behavior of the fluid. n<1 is the case of shear thinning (pseudo-plastic) fluids. The resistance of the fluid against the shear rate (viscosity of the fluid) decreases with increased shear rate. n=1 is the case of Newtonian fluids. In this case the viscosity of the fluid is independent of the imposed deformation rate. Most of the fluids are categorized in these two cases (Newtonian and shear thinning fluids). However, some of the fluids have a different behavior and are called shear thickening (dilatant) fluids. For shear thickening fluids n is greater than one. The viscosity of this type of fluids increases with an increase in the shear rate [23, 28].

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