

## Entropy generation calculation for laminar fully developed forced flow and heat transfer of nanofluids inside annuli

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### ABSTRACT

In this paper, second law analysis for calculations of the entropy generation due to the flow and heat transfer of water- $\text{Al}_2\text{O}_3$  and ethylene glycol- $\text{Al}_2\text{O}_3$  nanofluids inside annuli is presented. The physical properties of the nanofluids are calculated using empirical correlations. Constant heat fluxes at inner surface of the annuli are considered and fully developed condition for fluid flow and heat transfer is assumed. The control volume approach is selected for calculation of the entropy generation. Total entropy generation for different values of the nanoparticles volume fractions at different geometrical ratios is obtained and compared with those of the base fluid. Also, the geometrical ratios at which the minimum entropy generation is achieved are presented. The results show that when the ratio of the annuli length to its hydraulic diameter ( $L/D_h$ ) exceeds some critical values, adding of the nanoparticles is not efficient. For each value of the nanoparticles concentration, there is a length ratio ( $L/D_h$ ) at which the entropy generation is minimized.

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### 1. Introduction.

There are various techniques for enhancing the heat transfer of fluids such as changing in geometry, boundary conditions, or enhancing thermal conductivity of the fluid. Since the thermal conductivity of the solids are typically higher than that of liquids, by suspending nano- or larger-sized solid particles in fluids, the thermal conductivity is increased.

Many attempts in this field have been made to formulate the effective thermal conductivity and dynamic viscosity of nanofluids appropriately [1-5]. The results

showed that suspending nanoparticles in the fluid improve thermal conductivity of the base fluid substantially. For example, Xuan and Li [1] presented a study on the thermal conductivity of a nanofluid consisting of copper nanoparticles. The measured data showed that adding 2.5-7.5% copper oxide nanoparticles to the water increases its conductivity by about 24-78%. Also, there are some experimental and numerical works about the flow and heat transfer characteristics of the nanofluids in the literature. Some researchers have considered the application of the nanofluids in annulus [6-8]. Abu-Nada [6] has studied  $\text{Al}_2\text{O}_3$ -water nanofluid flow in an annulus using single phase approach. Different viscosity and thermal conductivity models are used to evaluate the

heat transfer enhancement in the annulus in his work. Izadi et al. [7] have also investigated the laminar forced convection of  $\text{Al}_2\text{O}_3$ -water nanofluid flow numerically in a two-dimensional axisymmetric flow inside an annulus with single phase approach.

Pressure drop developed during the flow of nanofluids is one of the important parameters determining the efficiency of nanofluids application. Pressure drop and pumping power are closely associated with each other. The physical properties that can influence the flow pressure drop are the nanofluids density and viscosity. It is expected that nanofluids with higher density and viscosity experience higher pressure drop. This has contributed to the disadvantages of nanofluids application as heat exchanging liquids. Lee et al. [9] and Yu et al. [10] investigated viscosity of water based  $\text{Al}_2\text{O}_3$  nanofluids and ethylene glycol based ZnO nanofluids. Their study clearly showed that the viscosity of nanofluids is higher than the base fluid. Vasu et al. [11] studied the thermal design of compact heat exchanger using nanofluids. In this study, it is found that adding 4% of  $\text{Al}_2\text{O}_3$  nanoparticles to water can double the pressure drop of water-  $\text{Al}_2\text{O}_3$  nanofluid flow. Pantzali et al. [12] reported substantial increases of nanofluids pressure drop and pumping power in plate heat exchanger. In that research, about 40% increase in pumping power was observed for nanofluids compared with water. They observed that for a given heat duty the required volumetric flow rates for both the water and the nanofluid are practically equal, while the necessary pumping power in the case of the nanofluid is up to two times higher than the corresponding value for water due to the higher kinematic viscosity of the fluid [12].

Singh et al. [13] provided a theoretical investigation of the entropy generation. They studied the alumina water nanofluid flow inside tubular microchannel (0.1 mm diameter), minichannel (1 mm diameter) and conventional channel (10 mm diameter). They found that it is not suitable to use alumina-water nanofluids with high viscosity in microchannels with laminar flow or minichannels and conventional channels with turbulent flow.

Moghaddami et al. [14] analytically examined the effects of adding nanoparticles on the entropy generation of the nanofluid flows through a circular pipe under uniform wall heat flux thermal boundary condition in both laminar and turbulent regimes. Based on the obtained results, they concluded that from the thermodynamic point of view, adding nanoparticles to the base fluid is efficient only when the heat transfer irreversibility is dominant. In their study, they assumed that the change in entropy of the nanofluid was a function of its pressure gradient. In classical thermodynamics, a nanofluid should be treated as

an incompressible fluid and its entropy change is only due to temperature difference.

Ijam and Saidur [15] analyzed a minichannel heat sink with SiC-water nanofluid and  $\text{TiO}_2$ -water nanofluid turbulent flow as coolants. They reported the maximum enhancement in heat flux by using  $\text{TiO}_2$ -water and SiC-water nanofluid at different fluid velocities and different nanoparticles volume fractions.

In the nanofluids flows, the improvement of the heat transfer properties causes the reduction in entropy generation. On the other hand, the increment in pressure drop gives more irreversibility and exergy loss in systems. Bejan [16] stated that when the entropy generation is minimized, the optimum design condition of a thermal system will be obtained. In the other words, the best design of heat exchangers is the one which includes considerations for how to increase heat transfer performance and reduce the pressure drop. Ko and Ting [17] have applied this concept to find the most appropriate flow conditions of a fully developed, laminar forced convection flow through a helical coil tube for which entropy generation is minimized. Ko [18] obtained the optimal mass flow rate for fully developed laminar forced convection in a helical coiled tube based on minimal entropy generation principle. Nag and Naresh [19] have studied second law optimization of convective heat transfer through a duct with constant heat flux.

Rafati et al. [20] studied the heat transfer performance of nanofluid in computer cooling systems. In their study, three nanoparticles of silica, alumina and titania were used, each with three different volumetric concentrations in the base fluid. The effect of the flow rate of nanofluid in the cooling process was investigated. Results of their experiments showed that there should be a balance between volumetric concentration of nanoparticles and the flow rate to satisfy the economy and power consumption of cooling system.

Mahian et al. [21] reviewed theoretical and computational contributions on entropy generation due to flow and heat transfer of nanofluids in different geometries and flow regimes. They tried to motivate the researchers to pay more attention to the entropy generation analysis of heat and fluid flow of nanofluids to improve the system performance. Also, Mahian et al. [22] presented an analytical solution which was presented on the entropy generation due to mixed convection between two isothermal cylinders where a transverse magnetic field was applied to the system.

Torabi and Aziz [23] investigated the entropy generation in an asymmetrically cooled hollow cylinder with temperature dependent thermal conductivity and internal heat generation. Their graphical results provide a comprehensive picture of how the local and total entropy

generation rates are affected by various thermal parameters and how the cooling conditions on the inside and the outside surfaces of the cylinder can affect the entropy generation.

Mahian et al. [24] analytically studied the entropy generation due to flow and heat transfer of nanofluids between co-rotating cylinders with constant heat flux on the walls. They found an optimum volume fraction of nanoparticles in which the average entropy generation is minimized.

There are also some research papers about natural convection heat transfer enhancement inside annuli using nanofluids [25, 26]. Forced convective heat transfer of nanofluids in a concentric annulus is investigated theoretically by Yang et al. [27]. They found that the Nusselt number has optimal bulk mean nanoparticle volume fraction value for alumina-water nanofluids, whereas it only increases monotonously with bulk mean nanoparticle volume fraction for titania-water nanofluids. Since, the annuli are one of the important geometries used in heat exchanging devices, it is essential to select proper values for nanoparticles concentration in such a manner that it gives the best second law efficiency. In this paper, the total irreversibility due to nanofluid flow and heat transfer in annuli is presented for different flow and thermal conditions. The optimum values for geometric ratios are obtained by analytical calculations. The effects of changing nanoparticles volume fraction and the base fluid Reynolds number on the pumping power and entropy generation are also considered.

## 2. Mathematical description of the problem and governing equations

### 2.1. Geometric configurations and problem description

Fig. 1 shows the geometric configurations of the problem under consideration. It consists of the steady, laminar forced convection and heat transfer of a nanofluid flowing inside an annulus.

Fluid passes through the annulus formed by concentric tubes and constant heat fluxes are applied at inner and

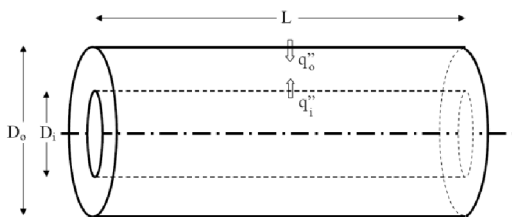


Figure 1. Geometric configuration of the concentric tube annulus

outer surface of the annulus. Thermally and hydrodynamically fully developed laminar flows inside the annulus are assumed for entropy generation calculation.

### 2.2. Thermophysical properties of the nanofluids

For small temperature differences, the thermophysical properties of the nanofluid are functions of nanoparticle volume concentration ( $\phi$ ), base fluid, and nanoparticles properties. Using the general formula for the mixtures, the following equation is used to evaluate the density of nanofluids

$$\rho_{nf} = (1 - \phi)\rho_{bf} + \phi\rho_p \tag{1}$$

where indices "p", "bf" and "nf" refer to particle, base fluid, and nanofluid respectively. Thermo physical properties of the base fluids and  $Al_2O_3$  nanoparticles are given in Table 1. As mentioned by Buongiorno [28], assuming that the nanoparticles and the base fluid are in thermal equilibrium, the nanofluid specific heat is derived from:

$$C_{p,nf} = \frac{(1 - \phi)\rho_{bf}C_{p,bf} + \phi\rho_p C_{p,p}}{\rho_{nf}} \tag{2}$$

Using experimental investigation, Pak and Cho [29] and Xuan and Roetzel [30] verified that the above equations are valid for nanofluid flows. Thermal conductivity and viscosity of the nanofluid can be obtained from the following equations which proposed by Wang et al. [31]. For water-  $Al_2O_3$  nanofluid they proposed

$$\mu_{nf} = (123\phi^2 + 7.3\phi + 1)\mu_{bf} \tag{3}$$

$$k_{nf} = (4.97\phi^2 + 2.72\phi + 1)k_{bf} \tag{4}$$

And for ethylene glycol-  $Al_2O_3$  nanofluid

$$\mu_{nf} = (306\phi^2 - 0.19\phi + 1)\mu_{bf} \tag{5}$$

$$k_{nf} = (28.905\phi^2 + 2.8273\phi + 1)k_{bf} \tag{6}$$

The above equations are valid when the temperature variation is smaller than  $10^\circ C$ .

The heat flux from each surface may be computed with expression of the form

$$q_i'' = h_i(T_{si} - T_m) \tag{7}$$

$$q''_0 = h_o(T_{so} - T_m) \tag{8}$$

The corresponding Nusselt numbers are of the form

$$Nu_i = \frac{h_i D_h}{k_{nf}} \tag{9}$$

$$Nu_o = \frac{h_o D_h}{k_{nf}} \tag{10}$$

where the hydraulic diameter  $D_h$  is

$$D_h = D_o - D_i \tag{11}$$

If uniform heat flux conditions exist at both surfaces, the Nusselt numbers may be computed from the following expressions [32]

$$Nu_i = \frac{Nu_{ii}}{1 - (q''_o / q''_i)\theta_i^*} \tag{12}$$

$$Nu_o = \frac{Nu_{oo}}{1 - (q''_o / q''_i)\theta_o^*} \tag{13}$$

For steady laminar flow in fully developed region, the influence coefficients  $Nu_{ii}$ ,  $Nu_{oo}$ ,  $\theta_i^*$  and  $\theta_o^*$  appearing in above equations can be obtained from Table 2 [32]. The friction factor for fully developed flow is as follows [32]:

$$f Re = \frac{16(1 - r^*)^2}{1 + r^* - 2r_m^*} \tag{14}$$

Table 1. Thermophysical properties of the base fluids and  $Al_2O_3$  nanoparticles at 300 K

Properties	Water	Ethylen - Glycol	Nanoparticle ( $Al_2O_3$ )
Density $\rho$ (kg/m <sup>3</sup> )	1000	1114.4	3900
Heat capacity $C_p$ (J/kg.K)	4180	2415	880
Thermal conductivity $k$ (W/mK)	0.6	0.252	40
Viscosity (Pa.s)	0.001	0.0157	--

where  $r^*$  is the radius ratio ( $r^* = r_i/r_o$ ) and  $r_m$  in the preceding equations is the radius where the maximum velocity occurs and  $r_m^*$  is its dimensionless form, which is defined as

$$r_m^* = \frac{r_m}{r_o} = \left(\frac{1 - r^{*2}}{2 \ln(1/r^*)}\right)^{1/2} \tag{15}$$

The pressure drop can be calculated using the following formula

$$\Delta P = (4f) \frac{L}{D_h} \frac{\rho_{nf} V^2}{2} \tag{16}$$

where  $V$  is the mean velocity of the nanofluid in this equation.

The first law of thermodynamics for steady state flow in a control volume is expressed as [33]

$$\frac{dQ}{dt} - \frac{dW}{dt} = \left(\frac{V_2^2}{2} + gz_2 + u_2 + \frac{p_2}{\rho}\right)(\dot{m}) - \left(\frac{V_1^2}{2} + gz_1 + u_1 + \frac{p_1}{\rho}\right)(\dot{m}) \tag{17}$$

Table 2. Influence coefficients for fully developed laminar flow in a circular tube annulus with uniform heat flux maintained at both surfaces [32].

$D_i/D_o$	$Nu_{ii}$	$Nu_{oo}$	$\theta_i^*$	$\theta_o^*$
0.0	—	4.364	$\infty$	0
0.05	17.81	4.792	2.18	0.0294
0.1	11.91	4.834	1.383	0.0562
0.2	8.499	4.833	0.905	0.1041
0.4	6.583	4.979	0.603	0.1823
0.6	5.912	5.099	0.473	0.2455
0.8	5.58	5.24	0.401	0.299
1.00	5.385	5.385	0.346	0.346

For an incompressible nanofluid, the internal energy ( $u$ ) is a function of its temperature ( $\Delta u = C\Delta T$ ). On the other hand for a control volume which consists of a horizontal annulus of length  $L$  with heat fluxes at inner and outer walls the above equation is reduced to

$$\int_0^L (q_i''r_i + q_o''r_o)2\pi dx = \dot{m}\left(\frac{P_2 - P_1}{\rho_{nf}}\right) + \dot{m}C_{nf}(T_2 - T_1) \quad (18)$$

where  $T_1$  and  $T_2$  are the fluid bulk temperature at the inlet and outlet of the control volume, respectively.

The second law of the thermodynamics for the mentioned control volume can be given as

$$\dot{S}_{gen} = \dot{m}(s_2 - s_1) - \int \frac{\delta\dot{Q}_i}{T_{si}} - \int \frac{\delta\dot{Q}_o}{T_{so}} \quad (19)$$

For an incompressible fluid

$$s_2 - s_1 = C_{nf} \ln\left(\frac{T_2}{T_1}\right) \quad (20)$$

therefore

$$\dot{S}_{gen} = \dot{m}C_{nf} \ln\left(\frac{T_2}{T_1}\right) - \int_0^L \frac{L\pi q_i'' D_i dx}{T_{si}} - \int_0^L \frac{L\pi q_o'' D_o dx}{T_{so}} \quad (21)$$

At each section  $T_{si}$  and  $T_{so}$  are obtained from eqs.7 and 8. For a given inlet conditions, the outlet temperature is calculated from the first law of thermodynamics. The entropy generation is determined by Eq. 21. All of the mentioned equations have been solved simultaneously using engineering equations solver (EES) code. Dimensions of the geometry, nanofluid volumetric flow rate, fluid properties, nanoparticles volume fraction and the values of heat fluxes at inner and outer surface of the annulus are specified as input parameters.

### 3. Validation of the method

In order to ensure the accuracy of the method, the results of the above method for entropy generation ratio is compared with those obtained for fully developed laminar flow in macro channel tube ( $D=10\text{mm}$  and  $L=1\text{m}$ ) by Singh et al. [13] in Fig. 2. All the input parameters considered in analysis are as those used by Singh et al. [13]. They study the water-  $\text{Al}_2\text{O}_3$  nanofluid and the

results are presented for a flow with Reynolds number of 1500. The entropy generation ratio is defined by

$$\text{Entropy Generation Ratio} = \frac{(\dot{S}_{gen})_{nf}}{(\dot{S}_{gen})_{bf}} \quad (22)$$

As can be seen, there is a good agreement between the results of above mentioned method and those obtained by Singh et.al [13]. Slight differences are due to that Singh et al. [13] used other equations for calculation of nanofluid viscosity and thermal conductivity instead of eqs. (3) and (4).

### 4. Results and discussion

First, the entropy generation inside an annulus with diameter ratio of 0.2 ( $D_i/D_o=0.2$ ) and hydraulic diameter of 0.008 ( $D_h=0.008$ ) is presented. The heat transfer rate of 197.5 Watt is applied at inner surface of the annulus. The bulk velocity of 0.125 m/s is selected to raise the base fluid flow temperature about 5 °K. The base fluid flow Reynolds number is 1000 ( $Re_{bf}=1000$ ).

Fig. 3 shows the entropy generation rate ratio against the length ratio ( $L/D_h$ ) of the annulus for different volume fractions of nanoparticles. These results are presented for water-  $\text{Al}_2\text{O}_3$  nanofluid. It must be noted that by increasing the aspect ratio ( $L/D_h$ ) a lower value of the heat flux at inner tube must be selected for a constant heat transfer rate and a constant hydraulic diameter. As can be seen, for the mentioned case, the minimum entropy generation ratio will be obtained at the length ratio of 1000. Also, by increasing the aspect ratio ( $L/D_h$ ), lower values of heat fluxes at the inner tube must be selected. As it can be seen, for the mentioned case, the minimum entropy generation ratio could be obtained at the length ratio of 1000. Also, it can be deduced that for larger aspect ratios than 10000 the

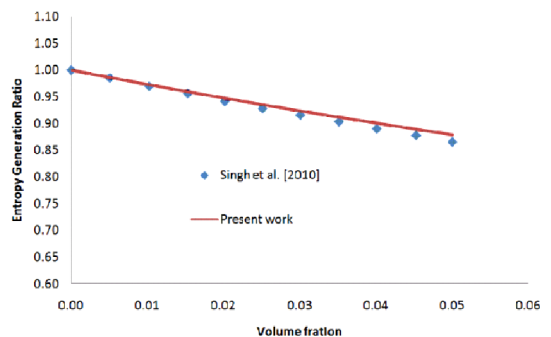


Fig. 2. Comparing the results of present study with other published data for laminar flow inside the tube.

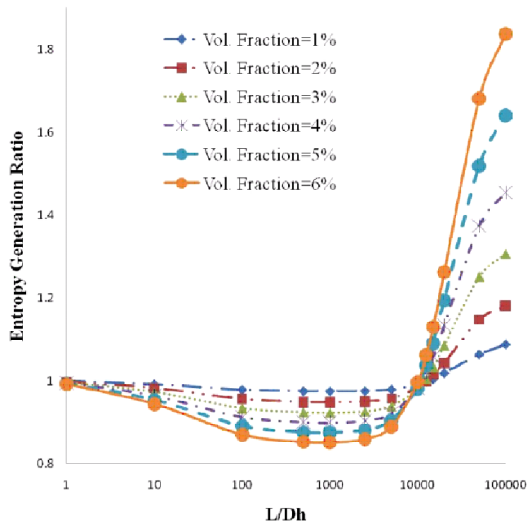


Fig. 3. Variations of the entropy generation ratio of water- $\text{Al}_2\text{O}_3$  nanofluid flow with the length ratio for different nanoparticles volume fraction

can be deduced that for larger aspect ratios than 10000 the use of nanofluid is not useful. In fact, for these ratios, the entropy generation due to pressure loss is dominant. For lower length ratios ( $L/D_h < 10000$ ), by increasing the nanoparticles volume fraction, better results can be achieved.

The variation of pumping power ratio with variation of nanoparticles volume fraction is shown in Fig. 4. The results are presented for the length ratio of 1000. It is evident that by adding 6.5% of the nanoparticles to the base fluid, the required pumping power is defined by:

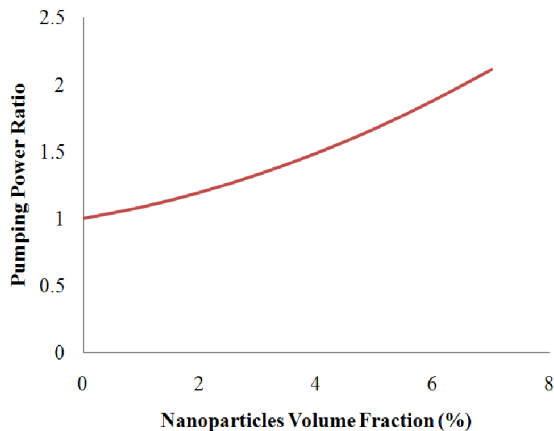


Fig. 4. Effects of nanoparticles volume fraction on the required pumping power

$$\text{pumping power ratio} = PP_{nf}/PP_{bf} \quad (23)$$

Results of the same study but with ethylene glycol- $\text{Al}_2\text{O}_3$  nanofluid are presented in Fig. 5. Since the viscosity of ethylene glycol is larger, the fluid flow irreversibility is too greater than water  $\text{Al}_2\text{O}_3$  nanofluid and use of nanofluids are advised only for the length ratios lower than 5000.

In Fig. 6, the entropy generation number ( $T_0 \dot{S}_{gen}/\dot{Q}$ ) against the nanoparticles volume fraction for mentioned nanofluids are compared. The results show that for the same amount of heat transfer, volumetric flow rate and the same geometry, by using ethylene glycol- $\text{Al}_2\text{O}_3$  nanofluid more irreversibility will occur.

The effects of diameter ratio on the entropy generation ratio are illustrated in Fig. 7. It must be noted that the diameters are selected to have a constant volumetric flow rate. These results are presented for 5% volume fraction of the  $\text{Al}_2\text{O}_3$  nanoparticles. For larger diameter ratios, the limits for length ratios at which the use of nanofluids are not acceptable will decrease. Also the maximum reduction in irreversibility by using the  $\text{Al}_2\text{O}_3$  nanofluid is limited to 12.5% in all cases. Variations of the entropy generation number ( $T_0 \dot{S}_{gen}/\dot{Q}$ ) by diameter ratio is shown in Fig. 8 for 5% volume fraction of nanoparticles and compared to

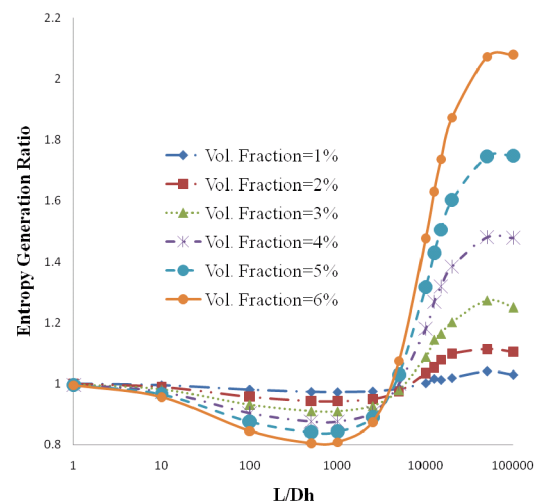


Fig. 5. Variations of the entropy generation ratio of ethylene glycol- $\text{Al}_2\text{O}_3$  nanofluid flow with the length ratio for different nanoparticles volume fraction

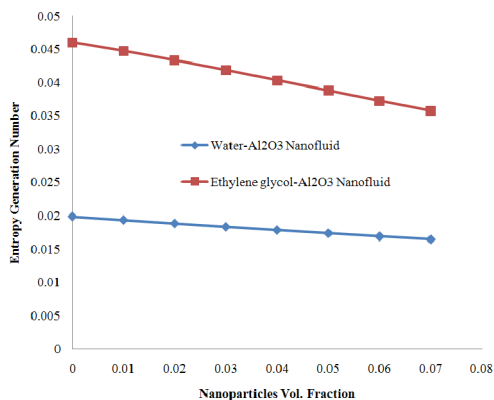


Fig. 6. Comparison between the entropy generation numbers of the water-Al<sub>2</sub>O<sub>3</sub> with ethylene glycol-Al<sub>2</sub>O<sub>3</sub> at the same condition.

the same results of the base fluid. These results are presented for length ratios of 500. As can be seen, for diameter ratios lower than 0.8 the use of nanofluid is efficient. The minimum entropy generation is obtained for diameter ratio of 0.8. At higher diameter ratios by adding the nanoparticles, higher pressure losses occur and the entropy generation will be augmented. It must be noted that  $D_i/D_o=1$  is not meaningful because in this case the cross section area of the annulus is zero. On the other hand, when the inside diameter approaches to outside diameter ( $D_i/D_o \rightarrow 1$ ), the cross section area of the annulus approaches to zero which means a very narrow passage that has high pressure drop. Therefore the entropy generation due to pressure drop will increase significantly. Dependency of the irreversibility to the base fluid flow Reynolds number is shown in Fig. 9. It should be noted that for changing the base fluid Reynolds number with the

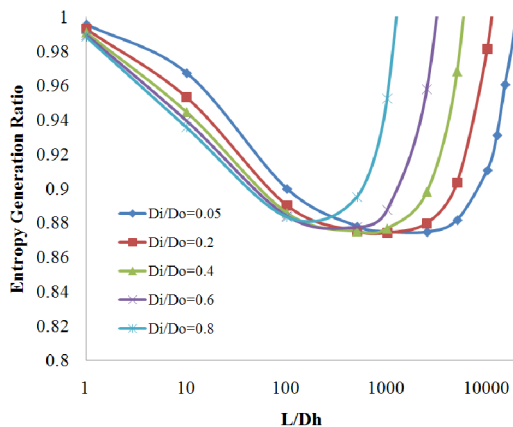


Fig.7. effects of diameter ratio on the entropy generation ratio of the water-Al<sub>2</sub>O<sub>3</sub> nanofluid flow.

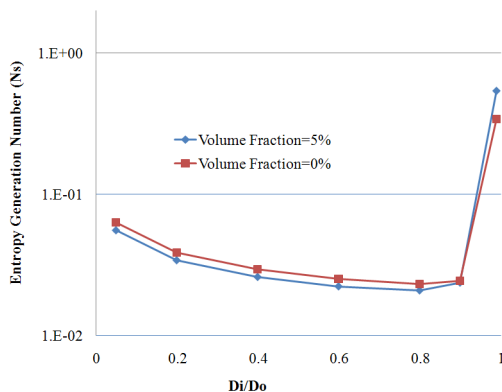


Fig.8. Variations of the entropy generation number ( $T_0 \dot{S}_{gen} / \dot{Q}$ ) with diameter ratio for length ratio of 500.

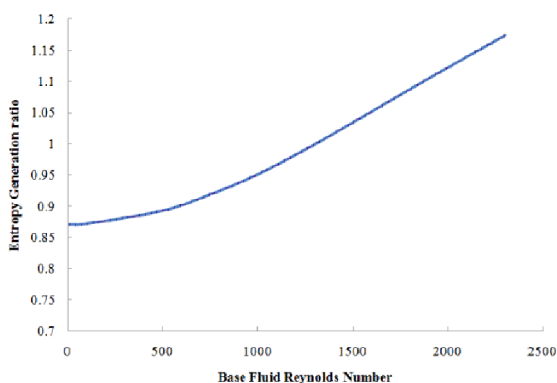


Fig. 9. Dependency of the irreversibility to the base fluid flow Reynolds number for:  $D_i/D_o=0.8$ ,  $L_e/D_h=1000$ ,  $\phi=5\%$  and  $Q^*=197.5$  Watt

same amount of volumetric flow rate, the velocity, inner and outer diameters should be simultaneously changed. These results are presented for diameter ratio of 0.8 and length ratio of 1000. The particles volume fraction of 5% is used for obtaining these results.

It is evident that for constant geometric ratios and heat transfer rates, the increase in Reynolds number will enhance the entropy generation ratio and at Reynolds number higher than 1250, addition of the nanoparticles are not acceptable. In fact for higher Reynolds numbers, adding the 5% of nanoparticles will result in higher viscosity and the entropy generation due to pressure drop will be dominant.

### 5. Conclusions

Results of the second law analysis for fully developed flow and heat transfer of the nanofluids inside annuli were presented in this paper. The more important results of analysis can be outlines as follows;

For a constant heat transfer rate, by increasing the length ratio of the annulus ( $L/D_h$ ), firstly the entropy generation ratio will decrease until reaches its minimum value. However for larger length ratios, there is an augmentation in entropy generation ratio and after a special value of the length ratios, the entropy generation of the nanofluid will be more than that of the base fluid. For very large length ratios, applying the nanoparticles is not advised.

The required pumping power is very sensitive to the nanoparticles volume fraction. For a mentioned case study, the required pumping power will be doubled if the volume fraction of the nanoparticles is 6.5%.

By applying large diameter ratios ( $D_i/D_o$ ), the applicability of the nanoparticles is limited for the annuluses with lower length ratios. This is due to the fact that for larger diameter ratios, the hydraulic diameter is lower and the irreversibility of the pressure loss can be dominant at lower length ratios.

At a constant heat transfer rate and for fixed values of the geometric ratios and volume fractions, by increasing the base fluid Reynolds number, the entropy generation will be raised.

## Nomenclature

$C_p$	Specific heat ( $\text{Jkg}^{-1}\text{K}^{-1}$ )
$D$	diameter (m)
$f$	friction factor (-)
$k$	Thermal conductivity ( $\text{Wm}^{-1}\text{K}^{-1}$ )
$L$	annulus length (m)
$m^{\bullet}$	mass flow rate ( $\text{kgs}^{-1}$ )
$N_s$	entropy generation number (-)
$Nu$	Nusselt number (-)
$P$	pressure (Pa)
$P.P.$	pumping power (W)
$Re$	Reynolds number (-)
$r$	Radius (m)
$T$	temperature (K)

## Greek symbols

$\phi$	nanoparticles volume fraction
$\mu$	viscosity (Pa.s)
$\rho$	density ( $\text{kgm}^{-3}$ )

## subscripts

$bf$	Base fluid
$gen$	generation

$i$	inside
$nf$	nanofluid
$m$	maiximum
$o$	outside
$p$	Nanoparticles

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